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LETTERS TO THE EDITOR

FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF A CLAMPED RECTANGULAR ORTHOTROPIC PLATE WITH A FREE-EDGE HOLE

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1. INTRODUCTION

Several papers deal with transverse vibrations of clamped isotropic plates with free-edge holes. An excellent survey of the field is presented in Leissa's classical treatise [1]. This is not the case when dealing with clamped, rectangular orthotropic plates. The present study tackles the title problem in the case of circular (Figure 1) and rectangular holes (see Figure 2). An approximate solution for the fundamental frequency coefficient is obtained by means of the optimized Rayleigh–Ritz method [2]. In the case of isotropic plates the results are in good engineering agreement with values available in the open literature [3].

2. APPROXIMATE SOLUTION

In the case of normal modes of vibration the amplitude $W(x, y)$ must comply with the governing functional [4]

$$J(W) = \iint_{\bar{P}} (D_1 W_{\bar{x}\bar{x}}^2 + 2D_1 \upsilon_2 W_{\bar{x}\bar{x}} W_{\bar{y}\bar{y}} + D_2 W_{\bar{y}\bar{y}}^2 + 4D_k W_{\bar{x}\bar{y}}^2)\, \mathrm{d}\bar{x}\, \mathrm{d}\bar{y} - \rho h \omega^2 \iint_{\bar{P}} W^2\, \mathrm{d}\bar{x}\, \mathrm{d}\bar{y} \quad (1)$$

and appropriate boundary conditions. Introducing the dimensionless variables

$$\bar{x} = ax, \qquad \bar{y} = by$$

and substituting in equation (1) one obtains

$$\frac{\lambda a^2}{D_1} J(W) = \iint_P (W_{xx}^2 + 2\upsilon_2 \lambda^2 W_{xx} W_{yy} + D_2' \lambda^4 W_{yy}^2 + 4D_k' \lambda^4 W_{xy}^2)\, \mathrm{d}x\, \mathrm{d}y - \Omega^2 \iint_P W^2 \mathrm{d}x\, \mathrm{d}y, \quad (2)$$

where $\lambda = a/b$, $D_2' = D_2/D_1$, $D_k' = D_k/D_1$, $\Omega^2 = (\rho h a^4/D_1)\omega^2$. In the case of an isotropic plate one has $D_1 = D_2 = D$, $\upsilon_2 = \upsilon$, $D_2' = 1$, $D_k' = (1 - \upsilon)/2$.

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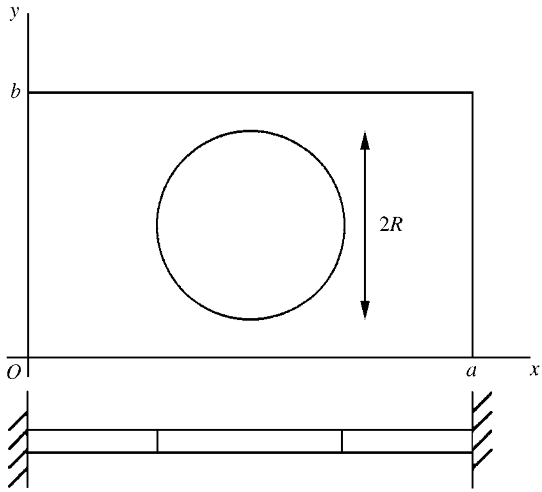

Figure 1. Clamped rectangular plate with a concentric, free-edge circular hole.

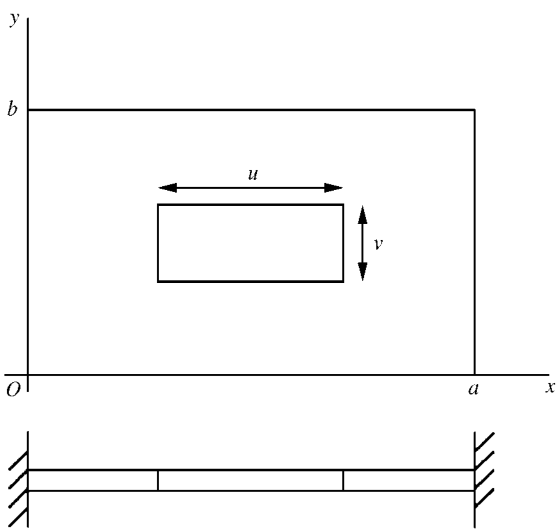

Figure 2. Clamped rectangular plate with a concentric, free-edge rectangular hole.

In order to apply the optimized Rayleigh–Ritz method the approximation

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(x, y) \tag{3}$$

is used, where

$$\varphi_j(x, y) = (x^{p+j-1} + a_{3j}x^3 + a_{2j}x^2)(y^{p+j-1} + a_{3j}y^3 + a_{2j}y^2), \tag{4}$$

and where the a_{3j}'s and a_{2j}'s are obtained by substituting each co-ordinate function in the governing essential conditions at the outer boundary and p is Rayleigh's optimization parameter.

Using Ritz' minimization condition one obtains

$$\frac{1}{2}\frac{\lambda a^2}{D_1}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{N}\left\{\iint_P [\varphi_{jxx}\varphi_{ixx} + \upsilon_2\lambda^2(\varphi_{jyy}\varphi_{ixx} + \varphi_{jxx}\varphi_{iyy}) + D'_2\lambda^4\varphi_{jyy}\varphi_{iyy} + 4D'_k\lambda^4\varphi_{jxy}\varphi_{ixy})]\,\mathrm{d}x\,\mathrm{d}y - \Omega^2\iint_D \varphi_j\varphi_i\,\mathrm{d}x\,\mathrm{d}y\right\} C_j = 0 \quad (5)$$

with $i = 1, 2, \ldots, N$. Equations (5) yields, finally, a determinantal equation whose lowest root is the fundamental frequency coefficient Ω_1. Minimizing Ω_1 with respect to the exponential parameter p one is able to optimize the value of Ω_1 [2].

3. NUMERICAL RESULTS

In the case of isotropic plates the Poisson ratio (v) has been taken equal to 0·30. When dealing with orthotropic plates the following set of parameters is used: $D'_2 = \frac{1}{2}$; $D'_k = \frac{1}{3}$; $\upsilon_2 = 0{\cdot}30$. The circular orifice and the rectangular hole† are assumed to be centrally located (Figures 1 and 2). The numerical determinations have been performed as a function of $r = 2R/b$ (Figure 1) and $\beta = v/b = u/b$ (Figure 2).

Tables 1 and 2 depict fundamental eigenvalues for circular and square holes, respectively, in the case of isotropic plates. Figure 3 shows a comparison of results with values recently obtained [3] in the case of clamped square plates with square holes. Good engineering agreement is achieved.

TABLE 1

Fundamental frequency coefficient of a isotropic clamped rectangular plate with a free-edge circular hole

λ	$r = 0$	$r = 0{\cdot}1$	$r = 0{\cdot}2$	$r = 0{\cdot}3$	$r = 0{\cdot}4$
1	36·000	36·279	37·940	40·273	45·927
1·5	60·847	61·251	62·886	65·266	70·975
2	98·522	99·154	100·415	103·592	109·039

TABLE 2

Fundamental frequency coefficient of a isotropic clamped rectangular plate with a free-edge square hole

λ	$\beta = 0$	$\beta = 0{\cdot}1$	$\beta = 0{\cdot}2$	$\beta = 0{\cdot}3$	$\beta = 0{\cdot}4$
1	36·000	36·245	38·123	40·732	49·311
1·5	60·847	61·217	63·237	66·426	73·365
2	98·522	99·109	100·387	104·382	111·696

† In the present study a square hole is considered; accordingly, $u/v = 1$.

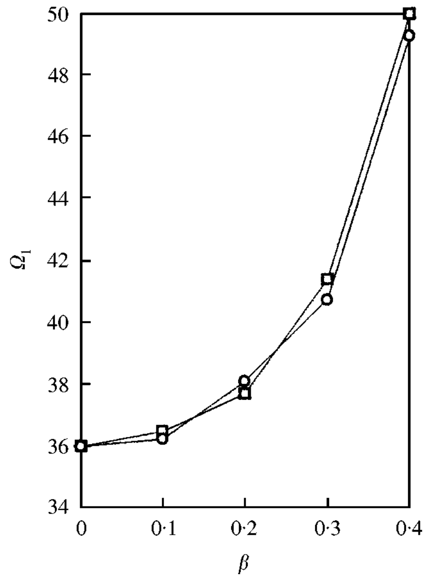

Figure 3. Fundamental frequency coefficient of a clamped, isotropic square plate with a central, free – edge square hole: comparison of results ($\beta = v/b$); □, reference [3]; ○, present study.

TABLE 3

Fundamental frequency coefficient of a orthotropic clamped rectangular plate with a free-edge circular hole

λ	$r = 0$	$r = 0{\cdot}1$	$r = 0{\cdot}2$	$r = 0{\cdot}3$	$r = 0{\cdot}4$
1	32·158	32·408	33·894	35·951	40·965
1·5	54·171	54·606	56·393	58·922	64·613
2	88·904	89·685	91·206	94·971	100·917

TABLE 4

Fundamental frequency coefficient of a orthotropic clamped rectangular plate with a free-edge square hole

λ	$\beta = 0$	$\beta = 0{\cdot}1$	$\beta = 0{\cdot}2$	$\beta = 0{\cdot}3$	$\beta = 0{\cdot}4$
1	32·158	32·378	34·057	36·374	43·928
1·5	54·171	54·569	56·769	60·123	66·846
2	88·904	89·630	91·172	95·848	103·559

Tables 3 and 4 deal with orthotropic plates. The variation of Ω_1 with holes size appears to be resonable when one considers the isotropic situation (Figure 3).

One concludes that for all the situations under analysis the fundamental frequency coefficient experiences a sustancial increment with the hole size. In other words the "dynamic stiffening" phenomenon takes place.

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